

Student Name: _____

Maths Class: _____

2022 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION



Mathematics Advanced

8 August 2022

**General
Instructions**

- Reading time – 10 minutes
- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In questions 11-35, show relevant mathematical reasoning and/or calculations

**Total marks:
100**

Section I – 10 marks (pages 1-5)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6-41)

- Attempt questions 11-35
- Allow about 2 hours and 45 minutes for this section

Marker's Use ONLY

MC	Q11-18	Q19-23	Q24	Q25-29	Q30-33	Q34-35
/10	/21	/19	/5	/20	/15	/10

Section I

10 marks

Attempt Questions 1-10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet at the back of the booklet for Questions 1-10.

1. What is equivalent to $\cos^2 3x$?
 - A. $1 + \sin^2 3x$
 - B. $1 - \sin^2 3x$
 - C. $\sin^2 3x - 1$
 - D. $-\sin^2 3x - 1$

2. What is the domain of $y = \ln(x - 1)$?
 - A. $x > 0$
 - B. $x \geq 1$
 - C. $x \geq -1$
 - D. $x > 1$

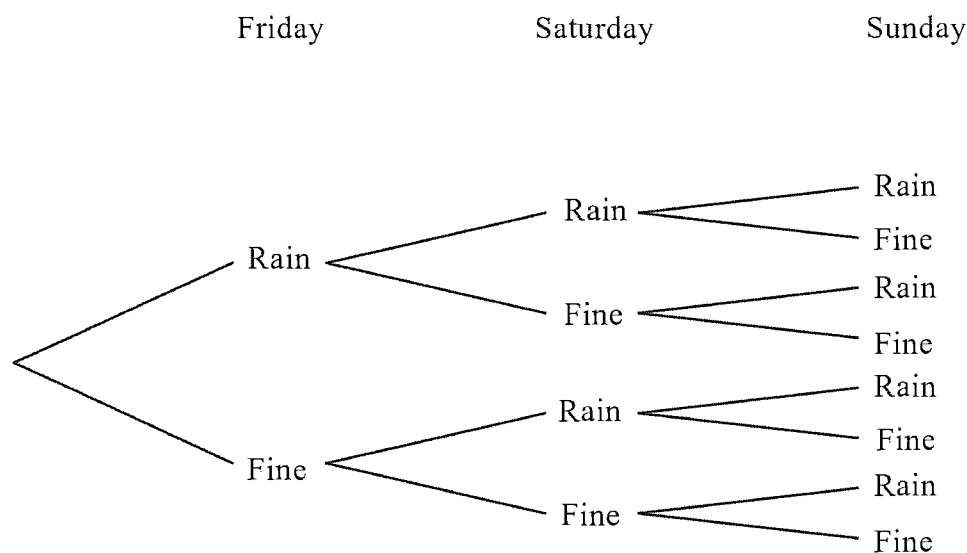
3. Given that $\sin \theta = \frac{3}{7}$ and $90^\circ < \theta < 180^\circ$, what is the value of $\tan \theta$?
 - A. $-\frac{3}{\sqrt{40}}$
 - B. $\frac{3}{\sqrt{40}}$
 - C. $-\frac{\sqrt{40}}{7}$
 - D. $\frac{\sqrt{40}}{7}$

4. What is the primitive of $x + \frac{1}{x}$?
 - A. $x^2 + 1 + C$
 - B. $\frac{x^2}{2} + 1 + C$
 - C. $\frac{x^2}{2} + \ln x + C$
 - D. $x^2 + \ln x + C$

5. The function $f(x) = 4(x + 1)^3$ is transformed by a horizontal translation 1 unit to the right, followed by a vertical dilation with scale factor 2. What is the equation of the new function $g(x)$?

- A. $8x^3$
- B. $2x^3$
- C. $8(x - 2)^3$
- D. $2(x + 2)^3$

6. Over the next three days, the chance that it rains on any given day is 0.91. What is the probability that it rains on at least one of the days?

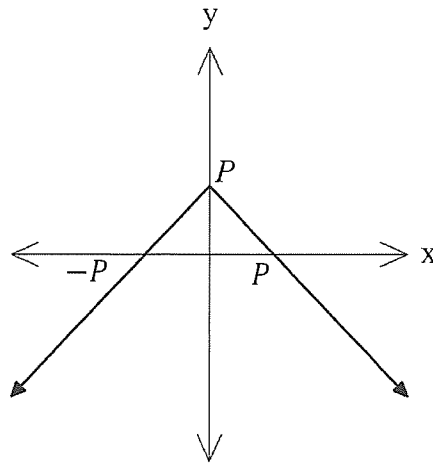


- A. 99.93%
- B. 75.36%
- C. 24.64%
- D. 0.07%

7. Given $y = 3^{2x+1}$, $\frac{dy}{dx} =$

- A. $2 \cdot 3^{2x+1}$
- B. $\ln 3 \cdot 3^{2x+1}$
- C. $2 \ln 3 \cdot 3^{2x+1}$
- D. 3^{2x+1}

8. The graph shows the function $y = P - |x|$.



Find the value of P , given:

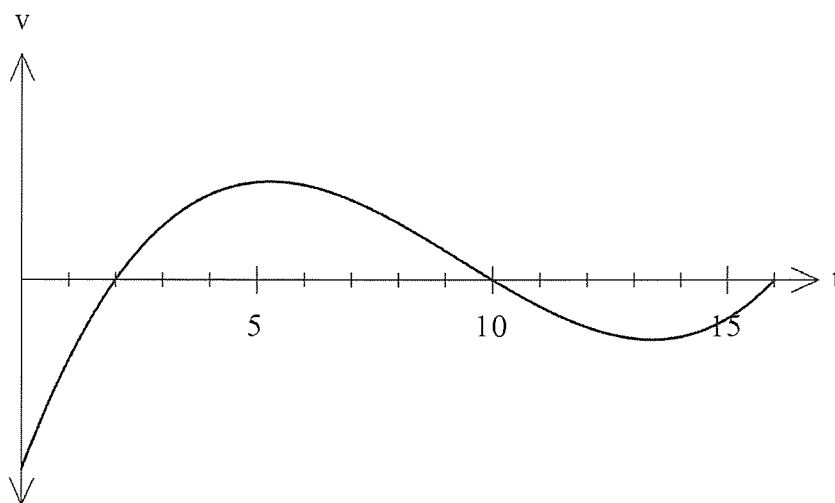
$$\int_{-P}^P (P - |x|) dx = 16$$

- A. $P = 4$
B. $P = 8$
C. $P = -4$
D. $P = -8$
9. Given that the mean of this dataset is 5.6, what is the value of k ?

<i>Score</i>	2	4	6	8
<i>Frequency</i>	2	k	2	8

- A. $k = 5$
B. $k = 6$
C. $k = 7$
D. $k = 8$

10. The graph below shows the velocity function of a particle during its travel for the first 16 seconds.



Given that the particle begins at the origin, which of the following statements is true?

- A. At $t = 2$, the particle is to the left of the origin, and travelling towards the origin.
- B. At $t = 5$, the particle is at the origin, and at rest.
- C. At $t = 10$, the particle is to the right of the origin, and at rest.
- D. At $t = 15$, the particle is at the origin, and travelling from left to right.

Please turn over for Multiple Choice answer sheet

Student Name: _____

Maths Class: _____

Mathematics Advanced

Section II Answer Booklet 1

90 marks

Attempt Questions 11-35

Allow about 2 hours and 45 minutes for this section

Booklet 1: Attempt Questions 11-24 (45) marks

Booklet 2: Attempt Questions 25-35 (45) marks

Instructions

- Write your Name and Maths Class at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 21-23. If you use this space, clearly indicate which question you are answering. You must also clearly indicate it on the original question.

Marker's Use ONLY

Q11-18	Q19-23	Q24
/21	/19	/5

Please turn over

Question 11 (2 marks)

Write down the exact value of $\tan \frac{7\pi}{6}$.

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Question 12 (2 marks)

Solve $|3x - 8| = 2$.

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Question 13 (3 marks)

Solve simultaneously

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$$y = x^2 + 1$$
$$y = 3 - x$$

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Please turn over for question 14

Question 14 (2 marks)

Solve:

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$$\frac{243}{6\sqrt{3} - \sqrt{27}} = 3^x$$

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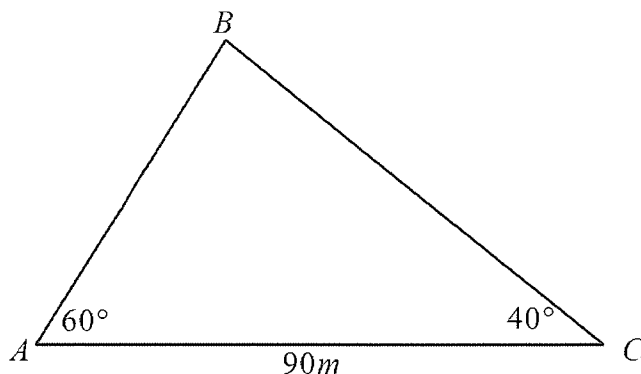
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Question 15 (5 marks)

Ant-man, Black Widow and Captain Marvel are positioned at points A, B and C as shown in the diagram below. Captain Marvel is 90m due east of Ant-man.



- (a) Find the bearing of Captain Marvel from Black Widow.

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- (b) Calculate the distance between Black Widow and Captain Marvel. Give your answer to the nearest metre.

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Question 15 continues over the page

(c) How far north is Black Widow from Captain Marvel? Answer to the nearest metre.

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Question 16 (4 marks)

(a) Differentiate $(\sin x)^3$

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(b) Hence find the gradient of the normal at $x = \frac{\pi}{4}$

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Question 17 (2 marks)

Find the derivative of

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$$\frac{e^{3x}}{x^2}$$

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Question 18 (1 mark)

Find the primitive of $\sqrt{5x + 1}$

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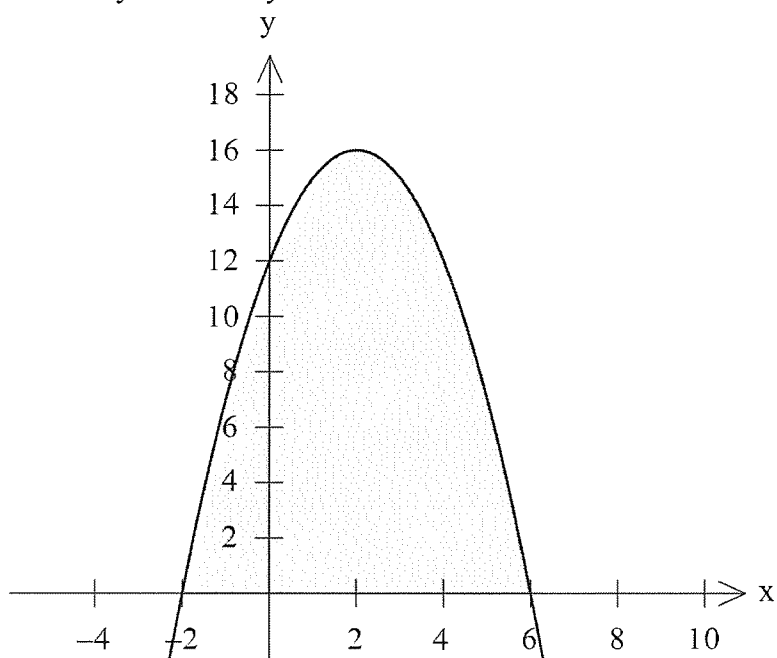
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Please turn over for question 19

Question 19 (5 marks)

Consider the region enclosed by the curve $y = 12 + 4x - x^2$ and the coordinate axes.



(a) Find the exact area of the region

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(b) Use the trapezoidal rule and 5 function values to estimate the area of the region.

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This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(c) Hence find the percentage error between the exact area and the estimation from part (b).

1

Please turn over for question 20

Question 20 (4 marks)

A tech company is interested in the relationship between how much time a person spends on their computer and how much time they spend on their phone. Their research team tracked the computer and phone usage of 10 people over the month of August. The average number of hours spent per day is recorded in the table below:

Person	A	B	C	D	E	F	G	H	J	K
Hours spent on a phone per day (x)	7.6	7.0	8.9	3.0	3.0	7.5	2.1	1.3	5.8	6.2
Hours spend on a computer per day (y)	1.7	1.1	0.7	5.8	5.2	1.7	6.9	7.1	3.3	4.1

- (a) Find the equation of the least-squares regression line. Give each coefficient correct to 3 significant figures.2

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- (b) By calculating Pearson’s correlation coefficient of this data, correct to 3 significant figures, what conclusion can be made about the data?2

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Question 21 (6 marks)

Consider the function

$$f(x) = \begin{cases} x^2 - 2x + 2 & -3 \leq x \leq 1 \\ x + 1 & 1 < x \leq 6 \end{cases}$$

(a) Justify why the function is NOT continuous at $x = 1$

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(b) Evaluate $f(2) - f(-2)$

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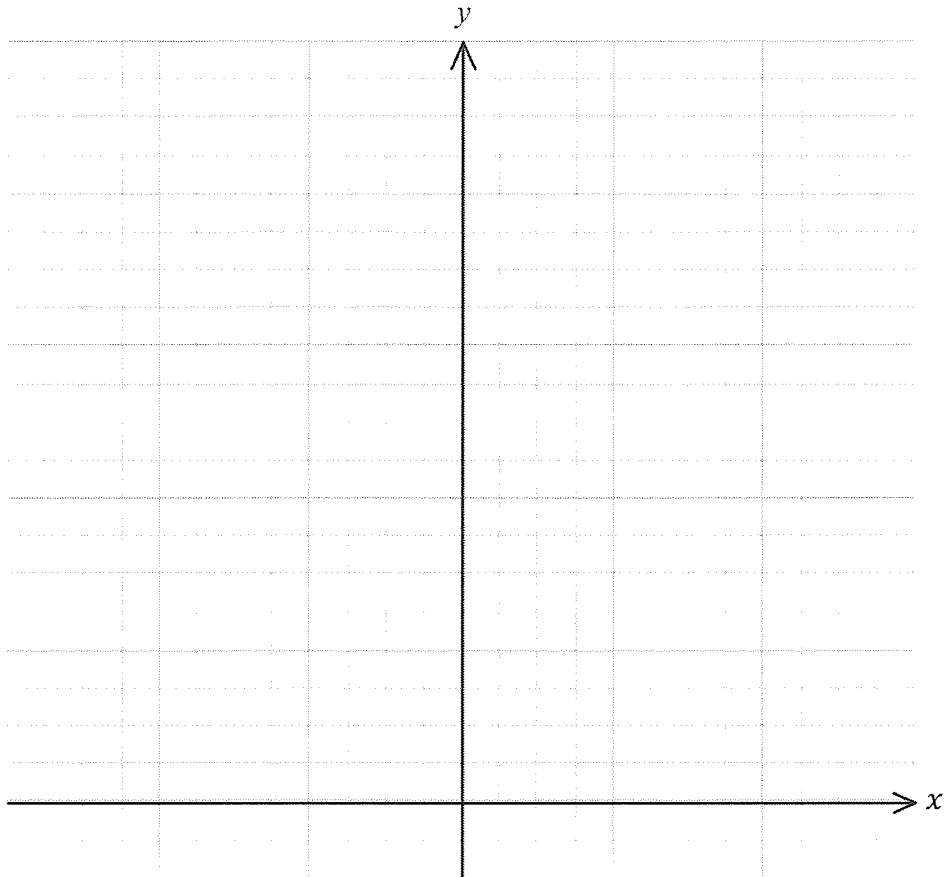
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(c) Sketch the function for the domain $-3 \leq x \leq 6$.

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Question 22 (2 marks)

Given $\frac{dy}{dx} = 1 - 6x + 3x^2$, find the equation of the curve given it passes through $(-2, 6)$

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Question 23 continues over the page

Question 23 (2 marks)

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A bag contains 4 red liquorice and 6 black liquorice lollies. Reese picks a lolly at random. If it is a red liquorice lolly, she will eat it and pick another. If it is a black liquorice lolly, she will replace it in the bag and choose another.

By drawing a tree diagram or otherwise, find the probability that after two draws, she chooses a red lolly and a black lolly, in any order.

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Question 24 (5 marks)

(a) Differentiate $y = x \ln x$

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(b) Hence find

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$$\int_e^{2e} \ln x \, dx$$

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Student Name: _____

Maths Class: _____

Mathematics Advanced

Section II Answer Booklet 2

Booklet 2: Attempt Questions 25-35 (45) marks

Instructions

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- Your responses should include relevant mathematical reasoning and/or calculations.
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Marker's Use ONLY

Q25-29	Q30-33	Q34-35
/20	/15	/10

Please turn over

Question 25 (2 marks)

In a circle, an arc length of 30cm subtends 80° at the centre of that circle. Find its radius. **2**

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Question 26 (3 marks)

Solve $\cos\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$ **3**

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Please turn over for Question 27

Question 27 (9 marks)

Consider the function $f(x) = 2x^2(k - x)$, where k is a constant.

- (a) If $f(x)$ has a maximum turning point at $(2, 8)$, show that $k = 3$.

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- (b) Find the minimum turning point (you do not need to test for this) and any inflexion points (prove by testing or otherwise).

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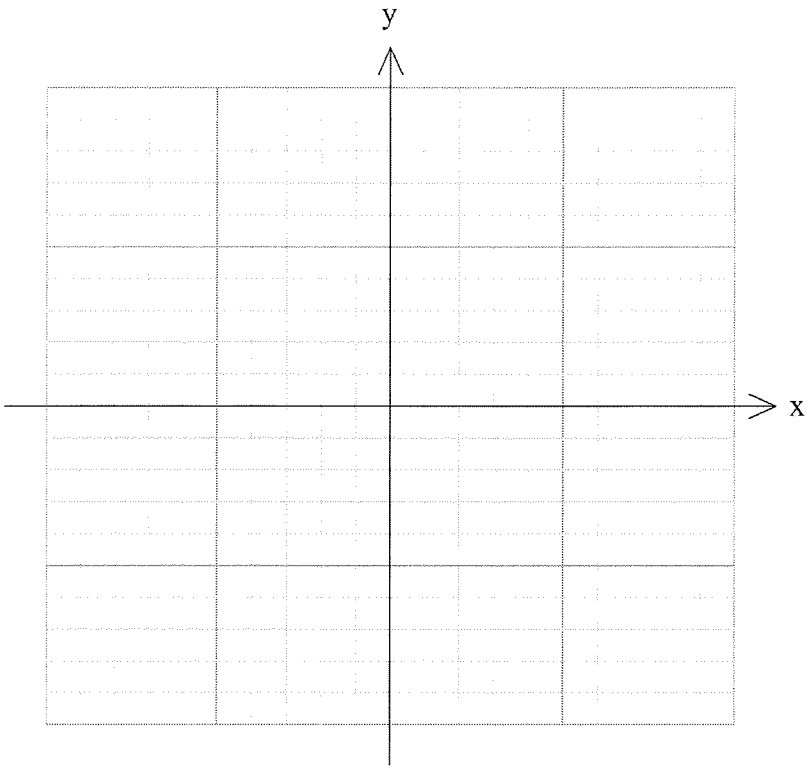
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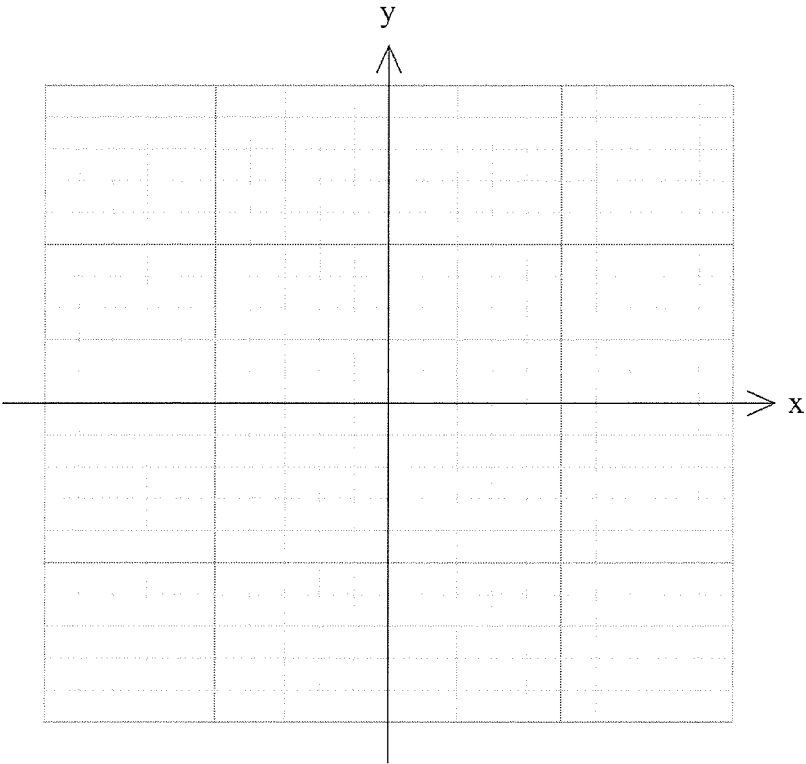
(c) Sketch the function $y = f(x)$, showing all features.

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(d) Hence sketch the function $y = -f(x) + 3$

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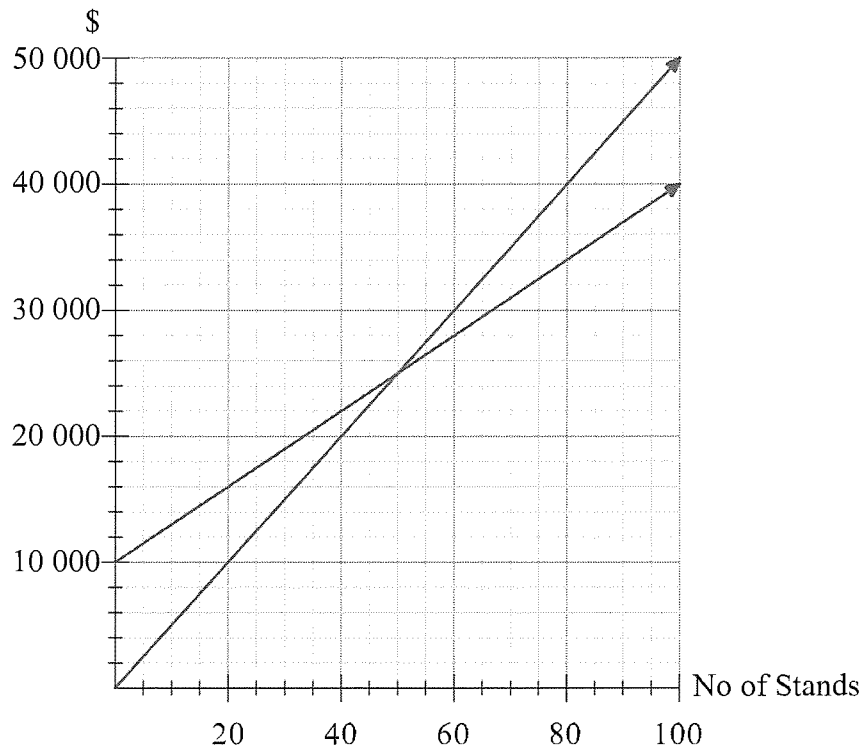


Please turn over for Question 28

Question 28 (2 marks)

The company, Upstanding Televisions, makes a television stand that sells for \$500. It costs \$300 to manufacture each unit and the machinery and warehouse costs \$10 000 per year.

This is shown in the graph below:



(a) How many stands must they sell to break even?

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(b) How much profit do they make if they sell 80 stands?

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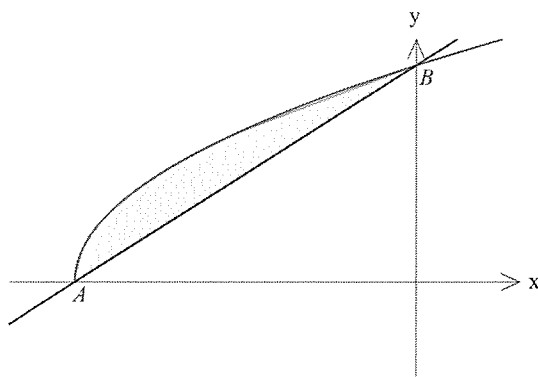
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Question 29 (4 marks)

In the following diagram, the curves $y = \sqrt{x+9}$ and the line $y = \frac{1}{3}x + 3$ intersect at A and B .

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Calculate the area bounded between the two curves.

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Please turn over for Question 30

Question 30 (2 marks)

A survey of where students choose to read the current news revealed that, on a regular basis, 33% read the news on Instagram, 45% read the news on Facebook and 44% do not read the news at all.

- (a) By constructing a Venn Diagram, or otherwise, find the percentage of students who get their news from both Facebook and Instagram. 1

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- (b) A student is selected at random. Given that this student does read the news on a regular basis, find the probability that this student gets their news from Facebook. 1

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Question 31 (5 marks)

A particle moves in a straight line such that at time t seconds, its distance x metres from a fixed point O of a line is given by the equation $x = 1 - \cos 2t$.

(a) State the period of the graph.

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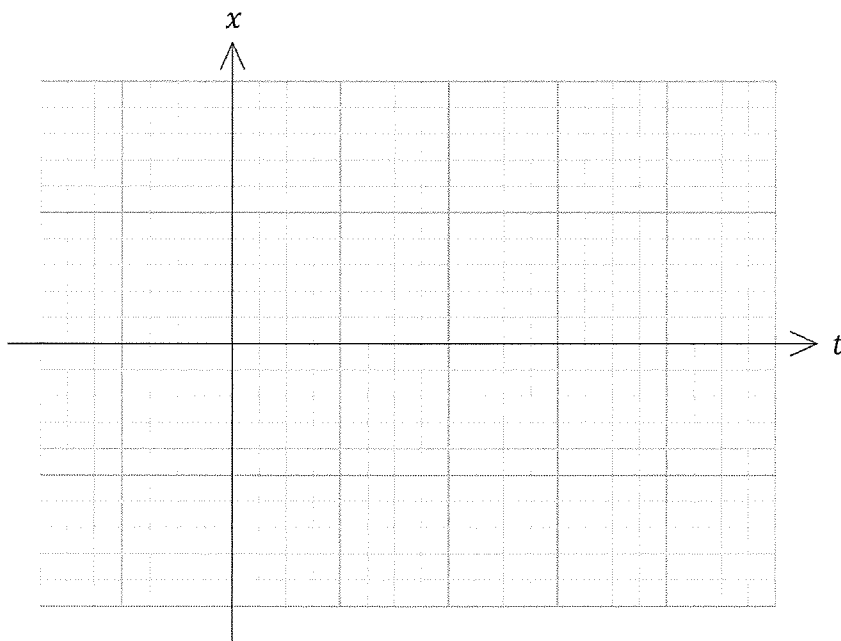
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(b) Sketch the graph for $0 \leq t \leq 2\pi$.

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(c) Hence state the times and positions where the particle is at rest in the first 3 seconds.

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Please turn over for Question 32

Question 32 (3 marks)

Rumour spreading at a school can be modelled by $N = 2 \times 1.61^{0.7t}$ where N is the number of students who have heard the rumour and t is the number of hours after it first started.

(a) How many students would have heard the rumour 6 hours after it started?

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(b) How long would it take all 165 Year 12 students to hear the rumour?

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Question 33 (5 marks)

Target Practice is a carnival game that involves throwing 3 tennis balls at a moving target. It generates the following probability distribution below, where X is the number of tennis balls that successfully hit the target.

x	0	1	2	3
$P(X = x)$	0.475	$2k^2$	0.025	$6k^2$

(a) Find the value of k . 2

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(b) Hence, find the expected number of tennis balls that will hit the target. 1

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(c) Mr Mosey runs the Target Practice stall at a carnival. He charges players \$3 per game. 2
However, for every tennis ball that hits the target, he pays out \$1 to players. By considering the profit he makes in each scenario, fill in the table below and find the expected profit Mr Mosey will make per player.

Profit (x)				
$P(\text{Profit} = x)$	0.475	$2k^2$	0.025	$6k^2$

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Please turn over for Question 34

Question 34 (4 marks)

The continuous random variable X has a probability density function

$$f(x) = \begin{cases} 4kx(2 - x^2) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the value of k .

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(b) Find the mode of the distribution to 2 decimal places.

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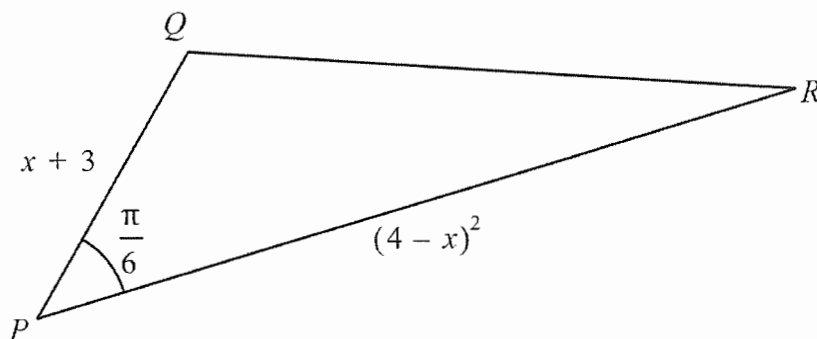
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Please turn over for Question 35

Question 35 (6 marks)

In $\triangle PQR$ the size of $\angle QPR$ is $\frac{\pi}{6}$. The length of PQ is $(x + 3)$ units and the length of PR is $(4 - x)^2$ units, as shown in the diagram.



- (a) Show that the area A of the triangle can be given by:

$$A = \frac{1}{4}(x^3 - 5x^2 - 8x + 48)$$

2

Student Name: Solutions

Maths Class: _____

2022 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

8 August 2022

Completely fill the response oval representing the most correct answer.

1. A ☐ B ☒ C ☐ D ☐
2. A ☐ B ☐ C ☐ D ☒
3. A ☒ B ☐ C ☐ D ☐
4. A ☐ B ☐ C ☒ D ☐
5. A ☒ B ☐ C ☐ D ☐
6. A ☒ B ☐ C ☐ D ☐
7. A ☐ B ☐ C ☒ D ☐
8. A ☒ B ☐ C ☐ D ☐
9. A ☐ B ☐ C ☐ D ☒
10. A ☐ B ☐ C ☒ D ☐

Student Name: _____

Maths Class: _____

Mathematics Advanced

Section II Answer Booklet 1

90 marks

Attempt Questions 11-35

Allow about 2 hours and 45 minutes for this section

Booklet 1: Attempt Questions 11-24 (45) marks

Booklet 2: Attempt Questions 25-35 (45) marks

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Instructions

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Marker's Use ONLY

Q11-18	Q19-23	Q24
/21	/19	/5

Please turn over

Question 11 (2 marks)

Write down the exact value of $\tan \frac{7\pi}{6}$.

$$\tan \frac{7\pi}{6} = -\tan \frac{\pi}{6} \\ = -\frac{1}{\sqrt{3}}$$

2

Question 12 (2 marks)

Solve $|3x - 8| = 2$.

$$3x - 8 = 2 \quad \text{or} \quad 3x - 8 = -2 \\ 3x = 10 \quad 3x = 6 \\ x = \frac{10}{3} \quad x = 2 \\ \therefore x = \frac{10}{3}, 2$$

2

Question 13 (3 marks)

Solve simultaneously

$$y = x^2 + 1 \quad \text{--- ①} \\ y = 3 - x \quad \text{--- ②}$$

sub ① $\rightarrow$ ②

$$x^2 + 1 = 3 - x$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = 1, -2$$

when $x = 1, y = 2$

$$x = -2, y = 5$$

3

Question 14 (2 marks)

Solve:

$$\frac{243}{6\sqrt{3} - \sqrt{27}} = 3^x$$

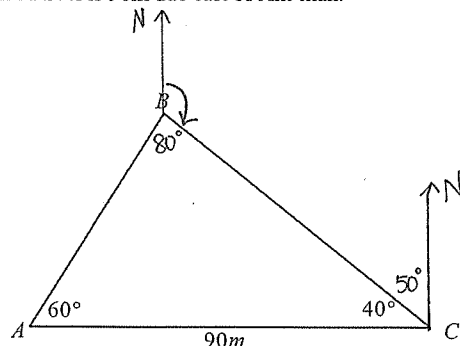
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$$\frac{243}{6\sqrt{3} - 3\sqrt{3}} \\ = \frac{3^5}{3\sqrt{3}} \\ = \frac{3^4}{\sqrt{3}} \\ = 3^3 \sqrt{3} \\ = 3^{3\frac{1}{2}} \\ \therefore x = 3\frac{1}{2}$$

Please turn over for question 14

Question 15 (5 marks)

Ant-man, Black Widow and Captain Marvel are positioned at points A, B and C as shown in the diagram below. Captain Marvel is 90m due east of Ant-man.



- (a) Find the bearing of Captain Marvel from Black Widow.

130°T or S50°E

- (b) Calculate the distance between Black Widow and Captain Marvel. Give your answer to the nearest metre.

$$\angle ABC = 80^\circ$$

$$\frac{90}{\sin 80^\circ} = \frac{BC}{\sin 60^\circ}$$

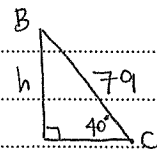
$$BC = \frac{90 \times \sin 60^\circ}{\sin 80^\circ}$$

$$= 79.14...$$

$$\approx 79 \text{ m}$$

Question 15 continues over the page

- (c) How far north is Black Widow from Captain Marvel? Answer to the nearest metre.



$$\sin 40^\circ = \frac{h}{79}$$

$$h = 79 \sin 40^\circ$$

$$= 50.78...$$

$$\approx 51 \text{ m}$$

OR

$$h = \frac{90 \sin 60^\circ}{\sin 80^\circ} \times \sin 40^\circ$$

$$= 50.87...$$

$$\approx 51 \text{ m}$$

Question 16 (4 marks)

- (a) Differentiate $(\sin x)^3$

$$3 \sin^2 x \cos x$$

- (b) Hence find the gradient of the normal at $x = \frac{\pi}{4}$

$$m_T = 3 \sin^2 \frac{\pi}{4} \cos \frac{\pi}{4}$$

$$= \frac{3}{2\sqrt{2}}$$

$$m_N = -\frac{2\sqrt{2}}{3}$$

$$\approx -0.9428...$$

Question 17 (2 marks)

Find the derivative of

$$u = e^{3x} \quad du = 3e^{3x} \quad \frac{e^{3x}}{x^2}$$

$$v = x^2 \quad dv = 2x$$

$$\frac{d}{dx} \left(\frac{e^{3x}}{x^2} \right) = \frac{3e^{3x} \times x^2 - 2xe^{3x}}{x^4} \quad \leftarrow \text{this is enough}$$

$$= \frac{xe^{3x}(3x-2)}{x^4}$$

$$= \frac{e^{3x}(3x-2)}{x^3}$$

Question 18 (1 mark)

Find the primitive of $\sqrt{5x+1}$

$$\int (5x+1)^{\frac{1}{2}} dx = \frac{(5x+1)^{\frac{3}{2}}}{5 \times \frac{3}{2}} + C$$

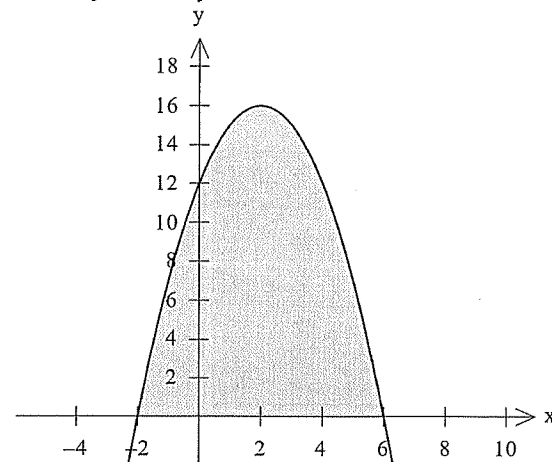
$$= \frac{2}{15} (5x+1)^{\frac{3}{2}} + C$$

Don't pencilise

Please turn over for question 19

Question 19 (5 marks)

Consider the region enclosed by the curve $y = 12 + 4x - x^2$ and the coordinate axes.



(a) Find the exact area of the region

$$\int_{-2}^6 (12 + 4x - x^2) dx = \left[12x + 2x^2 - \frac{x^3}{3} \right]_{-2}^6$$

$$= 12 \times 6 + 2 \times 6^2 - \frac{6^3}{3} - \left(12 \times (-2) + 2 \times (-2)^2 - \frac{(-2)^3}{3} \right)$$

$$= 72 - \left(-\frac{40}{3} \right)$$

$$= \frac{256}{3}$$

$$= 85 \frac{1}{3}$$

(b) Use the trapezoidal rule and 5 function values to estimate the area of the region.

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x	-2	0	2	4	6
$f(x)$	0	12	16	12	0

$$h=2$$

$$\therefore A \approx \frac{2}{2} (0+0+2(12+16+12))$$

$$\approx 80$$

(c) Hence find the percentage error between the exact area and the estimation from part (b).

1

$$\text{Error} : 85\frac{1}{3} - 80$$

$$= 5\frac{1}{3}$$

$$\frac{5\frac{1}{3}}{85\frac{1}{3}} \times 100\% = 6.25\%$$

Underestimated by 6.25%

Please turn over for question 20

Question 20 (4 marks)

A tech company is interested in the relationship between how much time a person spends on their computer and how much time they spend on their phone. Their research team tracked the computer and phone usage of 10 people over the month of August. The average number of hours spent per day is recorded in the table below:

Person	A	B	C	D	E	F	G	H	J	K
Hours spent on a phone per day (x)	7.6	7.0	8.9	3.0	3.0	7.5	2.1	1.3	5.8	6.2
Hours spend on a computer per day (y)	1.7	1.1	0.7	5.8	5.2	1.7	6.9	7.1	3.3	4.1

(a) Find the equation of the least-squares regression line. Give each coefficient correct to 3 significant figures.

2

$$A = 8.3694...$$

$$\approx 8.37 \text{ (3 s.f.)}$$

$$B = -0.87966...$$

$$\approx -0.880 \text{ (3 s.f.)}$$

$$\therefore y = 8.37 - 0.880x$$

(b) By calculating Pearson's correlation coefficient of this data, correct to 3 significant figures, what conclusion can be made about the data?

2

$$r = -0.96987...$$

$$\approx -0.970 \text{ (3 s.f.)}$$

Strong negative correlation

OR

The more time a person spends on the phone, the less time they spend on their computer.

Question 21 (6 marks)

Consider the function

$$f(x) = \begin{cases} x^2 - 2x + 2 & -3 \leq x \leq 1 \\ x + 1 & 1 < x \leq 6 \end{cases}$$

- (a) Justify why the function is NOT continuous at $x = 1$

1

$$L^- = 1 - 2 + 2$$
$$= 1$$

$$L^+ = 1 + 1$$
$$= 2$$

$$\therefore L^- \neq L^+ \therefore \text{not continuous}$$

- (b) Evaluate $f(2) - f(-2)$

2

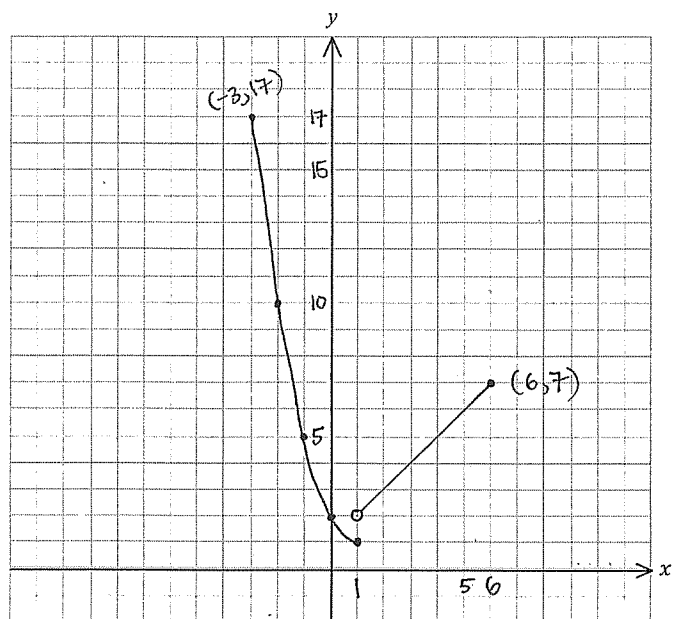
$$f(2) = 3$$

$$f(-2) = 10$$

$$\therefore 3 - 10 = -7$$

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(c) Sketch the function for the domain $-3 \leq x \leq 6$.



Question 22 (2 marks)

Given $\frac{dy}{dx} = 1 - 6x + 3x^2$, find the equation of the curve given it passes through $(-2, 6)$

$$y = x - 3x^2 + x^3 + C$$

at $(-2, 6)$

$$6 = -2 - 3 \times 2^2 + -2^3 + C$$

$$C = 28$$

$$\therefore y = x - 3x^2 + x^3 + 28$$

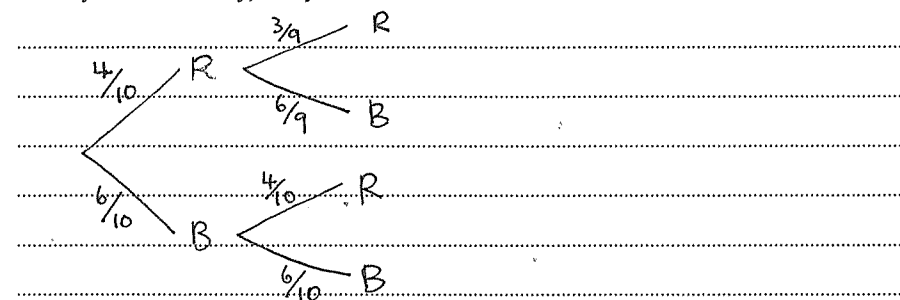
Question 23 continues over the page

3

Question 23 (2 marks)

A bag contains 4 red liquorice and 6 black liquorice lollies. Reese picks a lolly at random. If it is a red liquorice lolly, she will eat it and pick another. If it is a black liquorice lolly, she will replace it in the bag and choose another.

By drawing a tree diagram or otherwise, find the probability that after two draws, she chooses a red lolly and a black lolly, in any order.



$$P(RB \text{ or } BR) = \frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{10}$$

$$= \frac{38}{75}$$

2

Question 24 (5 marks)

(a) Differentiate $y = x \ln x$

$$\begin{aligned}\frac{dy}{dx} &= \ln x + x \times \frac{1}{x} \\ &= \ln x + 1\end{aligned}$$

(b) Hence find

$$\int_e^{2e} \ln x \, dx$$

If $\frac{d}{dx} x \ln x = \ln x + 1$

$$\int_e^{2e} \frac{d}{dx} x \ln x \, dx = \int_e^{2e} \ln x \, dx + \int_e^{2e} 1 \, dx$$

$$[x \ln x]_e^{2e} = \int_e^{2e} \ln x \, dx + [x]_e^{2e}$$

$$\therefore \int_e^{2e} \ln x \, dx = [x \ln x]_e^{2e} - [x]_e^{2e}$$

$$= 2e \ln 2e - e \ln e - (2e - e)$$

$$= 2e \ln 2e - e \ln e - e \leftarrow \text{sufficient}$$

$$= 2e \ln 2e - e - e$$

$$= 2e \ln 2e - 2e$$

$$= 2e (\ln 2 + \ln e) - 2e$$

$$= 2e \ln 2$$

End of Booklet 1

EXTRA PAPER - CLEARLY INDICATE WHICH QUESTION YOU ARE ANSWERING

2

3

Student Name: _____

Maths Class: _____

Mathematics Advanced

Section II Answer Booklet 2

Booklet 2: Attempt Questions 25-35 (45) marks

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Instructions

- Write your Name and Maths Class at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 39-41. If you use this space, clearly indicate which question you are answering. You must also clearly indicate it on the original question.

Marker's Use ONLY		
Q25-29	Q30-33	Q34-35
/20	/15	/10

Please turn over

Question 25 (2 marks)

In a circle, an arc length of 30cm subtends 80° at the centre of that circle. Find its radius.

2

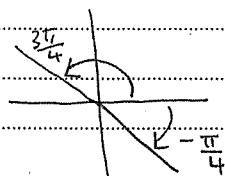
$$\begin{aligned} 30 &= \frac{80}{360} \times 2\pi r \\ 30 &= \frac{4}{9}\pi r \\ \frac{30 \times 9}{4} &= \pi r \\ \pi r &= \frac{135}{2} \\ r &= \frac{135}{2\pi} \\ &\approx 21.5 \text{ cm (1 d.p.)} \end{aligned}$$

Question 26 (3 marks)

Solve $\cos\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$

3

$$\begin{aligned} 0 &\leq x \leq 2\pi \\ 0 &\leq \frac{x}{2} \leq \pi \\ -\frac{\pi}{4} &\leq \frac{x}{2} - \frac{\pi}{4} \leq \frac{3\pi}{4} \\ \cos\left(\frac{x}{2} - \frac{\pi}{4}\right) &= \frac{1}{2} \\ \frac{x}{2} - \frac{\pi}{4} &= \frac{\pi}{3} \text{ only} \\ \frac{x}{2} &= \frac{7\pi}{12} \\ x &= \frac{7\pi}{6} \end{aligned}$$



Please turn over for Question 27

Question 27 (9 marks)

Consider the function $f(x) = 2x^2(k - x)$, where k is a constant.

(a) If $f(x)$ has a maximum turning point at $(2, 8)$, show that $k = 3$.

2

$$\begin{aligned} f'(2) &= 0 & f'(x) &= 4kx - 6x^2 & \text{OR} & & f(2) &= 8 \\ 0 &= 4k \times 2 - 6 \times 2^2 & & & & & 8 &= 2 \times 2^2(k-2) \\ 0 &= 2k - 6 & & & & & 8 &= 8(k-2) \\ 2k &= 6 & & & & & 1 &= k-2 \\ k &= 3 & & & & & k &= 3 \end{aligned}$$

(b) Find the minimum turning point (you do not need to test for this) and any inflexion points (prove by testing or otherwise).

3

$$\begin{aligned} f'(x) &= 12x - 6x^2 \\ f'(x) &= 6x(2 - x) \\ \text{since } (2, 8) &\text{ is max} \\ 6x &= 0 \Rightarrow x = 0 \text{ } y = 0 \text{ is min.} \\ f''(x) &= 12 - 12x \\ \text{Possible inflexion at } f''(x) &= 0 \\ 0 &= 12 - 12x \\ x &= 1 \end{aligned}$$

Since $x=1$ is between $(2, 8)$ max turning and $(0, 0)$ which is min turning point $(1, 4)$ is an inflexion point

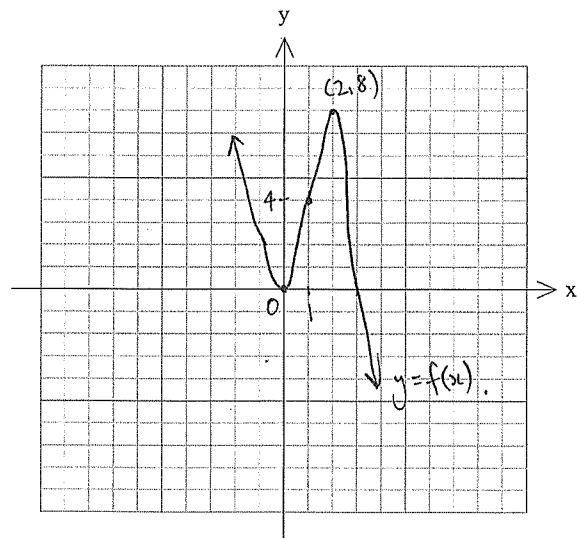
OR

x	0	1	2
$f''(x)$	12	0	-12
	U		N

Concavity changes: $(1, 4)$ is an inflexion point.

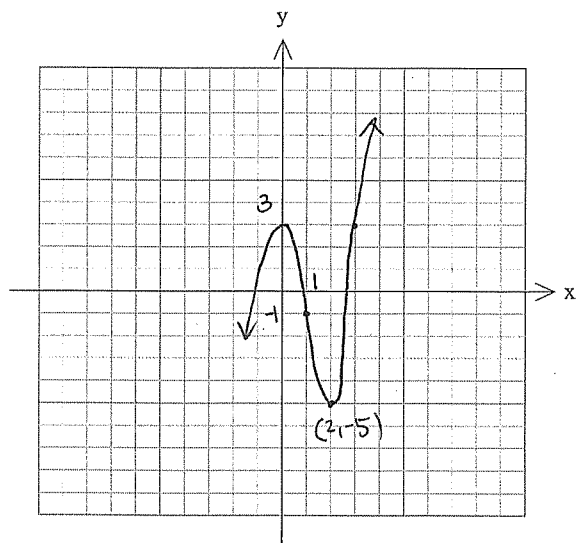
(c) Sketch the function $y = f(x)$, showing all features.

2



(d) Hence sketch the function $y = -f(x) + 3$

2

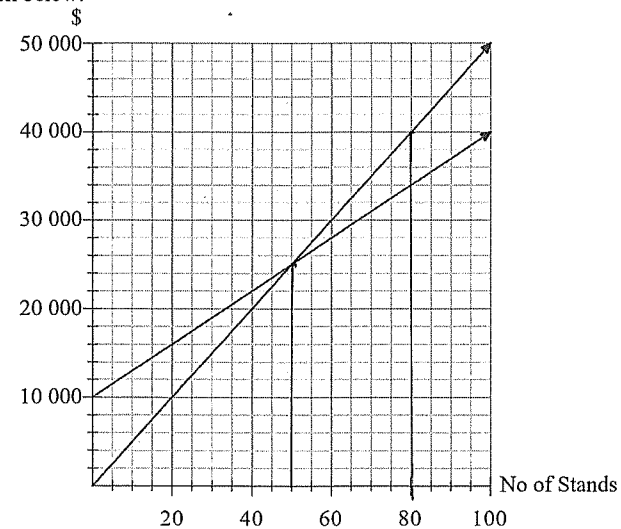


Please turn over for Question 28

Question 28 (2 marks)

The company, Upstanding Televisions, makes a television stand that sells for \$500. It costs \$300 to manufacture each unit and the machinery and warehouse costs \$10 000 per year.

This is shown in the graph below:



(a) How many stands must they sell to break even?

1

50 stands

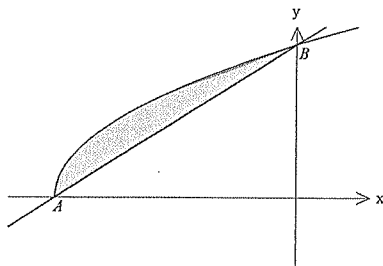
(b) How much profit do they make if they sell 80 stands?

1

$40\,000 - 34\,000 = 6\,000$
\$6000 profit

Question 29 (4 marks)

In the following diagram, the curves $y = \sqrt{x+9}$ and the line $y = \frac{1}{3}x + 3$ intersect at A and B.



Calculate the area bounded between the two curves.

$$A: (-9, 0) \quad B(0, 3)$$

$$\text{Area} = \int_{-9}^0 \sqrt{x+9} - \left(\frac{1}{3}x + 3\right) dx$$

$$= \int_{-9}^0 (x+9)^{\frac{1}{2}} - \frac{1}{3}x - 3 dx$$

$$= \left[\frac{(x+9)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{6}x^2 - 3x \right]_{-9}^0$$

$$= \frac{9^{\frac{3}{2}}}{\frac{3}{2}} - 0 - 0 - \left(0 - \frac{1}{6} \times 9^2 - 3 \times -9\right)$$

$$= 18 - \frac{27}{2}$$

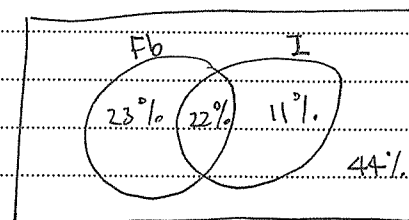
$$= 4.5 \text{ u}^2$$

Please turn over for Question 30

Question 30 (2 marks)

A survey of where students choose to read the current news revealed that, on a regular basis, 33% read the news on Instagram, 45% read the news on Facebook and 44% do not read the news at all.

- (a) By constructing a Venn Diagram, or otherwise, find the percentage of students who get their news from both Facebook and Instagram.



$$P(Fb \cap Insta) = 22\%$$

- (b) A student is selected at random. Given that this student does read the news on a regular basis, find the probability that this student gets their news from Facebook.

$$P(Fb | News) = \frac{P(Fb \cap News)}{P(News)}$$

$$= \frac{23 + 22}{23 + 22 + 11}$$

$$= \frac{45}{56} \approx 80.4\% \text{ (1 d.p.)}$$

Question 31 (5 marks)

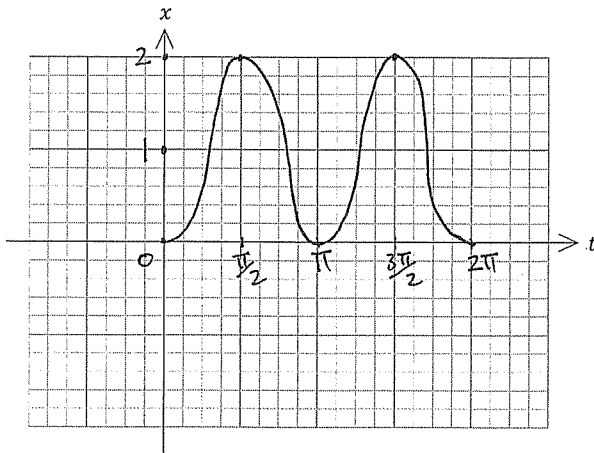
A particle moves in a straight line such that at time t seconds, its distance x metres from a fixed point O of a line is given by the equation $x = 1 - \cos 2t$.

(a) State the period of the graph.

$$T = \frac{2\pi}{2}$$

$$= \pi$$

(b) Sketch the graph for $0 \leq t \leq 2\pi$.



(c) Hence state the times and positions where the particle is at rest in the first 3 seconds.

$$t = 0 \text{ or } t = \frac{\pi}{2} \text{ seconds}$$

Please turn over for Question 32

Question 32 (3 marks)

Rumour spreading at a school can be modelled by $N = 2 \times 1.61^{0.7t}$ where N is the number of students who have heard the rumour and t is the number of hours after it first started.

(a) How many students would have heard the rumour 6 hours after it started?

$$N = 2 \times 1.61^{0.7 \times 6}$$

$$= 14.78 \dots$$

$$\div 14 \text{ students or } 15 \text{ students}$$

(both accepted)

(b) How long would it take all 165 Year 12 students to hear the rumour?

$$165 = 2 \times 1.61^{0.7t}$$

$$\frac{165}{2} = 1.61^{0.7t}$$

$$\ln\left(\frac{165}{2}\right) = \ln(1.61^{0.7t})$$

$$\ln\left(\frac{165}{2}\right) = 0.7t (\ln 1.61)$$

$$t = \frac{\ln\left(\frac{165}{2}\right)}{0.7 \ln 1.61}$$

$$= 13.23 \dots$$

$$\div 13.24 \text{ hours}$$

Question 33 (5 marks)

Target Practice is a carnival game that involves throwing 3 tennis balls at a moving target. It generates the following probability distribution below, where X is the number of tennis balls that successfully hit the target.

x	0	1	2	3
$P(X = x)$	0.475	$2k^2$	0.025	$6k^2$

(a) Find the value of k .

2

$$0.475 + 0.025 + 2k^2 + 6k^2 = 1$$

$$8k^2 = 0.5$$

$$k^2 = \frac{1}{16}$$

$$k = \pm \frac{1}{4}$$

(b) Hence, find the expected number of tennis balls that will hit the target.

1

$$E(X) = 1.3$$

(c) Mr Mosey runs the Target Practice stall at a carnival. He charges players \$3 per game.

2

However, for every tennis ball that hits the target, he pays out \$1 to players. By considering the profit he makes in each scenario, fill in the table below and find the expected profit Mr Mosey will make per player.

Profit (x)	\$3	\$2	\$1	\$0
$P(\text{Profit} = x)$	0.475	$2k^2$	0.025	$6k^2$

$$E(X) = \$1.70$$

Please turn over for Question 34

Question 34 (4 marks)

The continuous random variable X has a probability density function

$$f(x) = \begin{cases} 4kx(2 - x^2) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the value of k .

2

$$\int_0^1 8kx - 4kx^3 dx = 1$$

$$[4kx^2 - kx^4]_0^1 = 1$$

$$4k - k - (0 - 0) = 1$$

$$3k = 1$$

$$k = \frac{1}{3}$$

(b) Find the mode of the distribution to 2 decimal places.

$$f(x) = \frac{8}{2}x - \frac{4}{2}x^3$$

$$f'(x) = \frac{8}{2} - 4x^2$$

Mode is a maximum turning point.

$$f'(x) = 0$$

$$0 = \frac{8}{2} - 4x^2$$

$$4x^2 = \frac{8}{2}$$

$$x^2 = \frac{2}{2}$$

$$x = \pm \sqrt{\frac{2}{2}}$$

$$\text{but } 0 < x < 1$$

$$\therefore x = \sqrt{\frac{2}{2}}$$

max as $f'(x)$ is upside down

parabola.

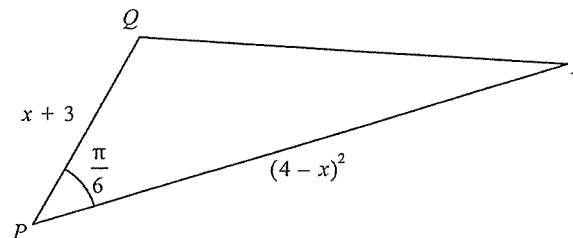
$$\therefore x = \sqrt{\frac{2}{2}} \text{ is mode}$$

$$\approx 0.82 \text{ (2 d.p.)}$$

Please turn over for Question 35

Question 35 (6 marks)

In $\triangle PQR$ the size of $\angle QPR$ is $\frac{\pi}{6}$. The length of PQ is $(x+3)$ units and the length of PR is $(4-x)^2$ units, as shown in the diagram.



(a) Show that the area A of the triangle can be given by:

$$A = \frac{1}{4}(x^3 - 5x^2 - 8x + 48)$$

$$A = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}(x+3)(4-x)^2 \sin \frac{\pi}{6}$$

$$= \frac{1}{2}(x+3)(16-8x+x^2) \times \frac{1}{2}$$

$$= \frac{1}{4}(16x-8x^2+x^3+48-24x+3x^2)$$

$$= \frac{1}{4}(x^3-5x^2-8x+48)$$

(b) Find the length of QR for which the area of ΔPQR is a maximum.

$$\frac{dA}{dx} = \frac{1}{4}(3x^2 - 10x - 8)$$

Maximum when $\frac{dA}{dx} = 0$

$$0 = \frac{1}{4}(3x^2 - 10x - 8)$$

$$0 = 3x^2 - 10x - 8$$

$$0 = (3x + 2)(x - 4)$$

$$x = -\frac{2}{3} \text{ or } x = 4$$

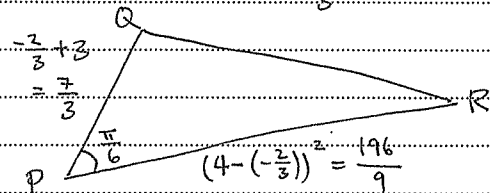
but as $(4-x)^2$ is a length, $4-x > 0$
 $x < 4$

$$\therefore x = -\frac{2}{3}$$

Test:

x	-1	$-\frac{2}{3}$	$-\frac{1}{3}$
$\frac{dA}{dx}$	$\frac{5}{4}$	0	$-\frac{13}{12}$

$\therefore$ Max at $x = -\frac{2}{3}$



$$\begin{aligned} QR^2 &= \left(\frac{7}{3}\right)^2 + \left(\frac{196}{9}\right)^2 - 2 \times \frac{7}{3} \times \frac{196}{9} \cos \frac{\pi}{6} \\ &= \frac{38857}{81} - \frac{2744}{27} \times \frac{\sqrt{3}}{2} \\ &= 391.70... \end{aligned}$$

$$QR = 19.79...$$

$$\doteq 19.8 \text{ (1 d.p.)}$$

End of paper